Denying the Liar Reaffirmed

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1.

In ‘Undeniably Paradoxical: Reply to Jacquette,’ John Barker offers a criticism of my recent *Polish Journal of Philosophy* essay, ‘Denying the Liar.’ If Barker is right, then I have correctly but unsurprisingly shown only that a construction other than the liar is not genuinely paradoxical, while leaving the liar paradox itself, properly so-called, completely untouched. Barker twice in the essay accuses my reasoning of being ‘flawed,’ but Barker’s own exposition and criticism is riddled from beginning to end with confusions and misunderstandings. Barker’s errors, as I show in what follows, concern not only my argument, but his interpretation and application of the classical literature and logical principles he cites in trying to refute my analysis of the liar. As usual, it takes more paper and ink to set these matters straight than to present the original argument or its criticism.

2.

First, contrary to Barker’s unsupported assertion, I do not claim to ‘solve’ the liar paradox by ‘simply declaring Liar sentences to be false.’ I do not try to solve the liar paradox at all, but rather to argue that there is no liar paradox to solve in the first place, merely a misperception that a genuine logical antinomy exists. Nor do I anywhere suggest that it is enough simply to declare that the liar sentence is false. If that were my strategy, then I suppose I could unhorse the problem even more directly simply by declaring that the liar is not a paradox. Rather, my point is that in contrast with the impression made by careless informal derivations of the liar paradox from the constructibility of the liar sentence, there is no formally valid derivation of the second horn of the liar paradox dilemma from the assumption that the liar sentence is false to the conclusion that therefore the liar sentence is true. Recognizing that the liar sentence is simply false
in failed efforts to derive the second paradox dilemma horn, instead of simply declaring that the liar sentence is false, demonstrates that the liar sentence as I formalize it does not support a genuine paradox in the sense of entailing an explicit syntactical inconsistency.

3.

The problem Barker raises has primarily to do with whether or not I have properly symbolized the liar sentence as a conditional rather than biconditional. I maintain that the liar sentence is a conditional that states of itself that it is false:

\[(L) \ L \rightarrow \text{FALSE}^{[L]}\]

I am aware that this account might strike some as unexpected, which is why I devote an Addendum in the original paper to discussing why the liar sentence cannot be biconditional if it is to constitute a nontrivial foundation for the liar paradox as an explicit logical contradiction.

\[(L^*) \ L^* \leftrightarrow \text{FALSE}^{[L^*]}\]

Barker writes: ‘Jacquette does consider a version of the Liar paradox based on (L*), but confines his discussion to an addendum.’ In view of my argument that the liar is correctly regarded as a conditional, it is natural, of course, that I should discuss the biconditional version only as an aside. After my refutation of the liar paradox, as I think it rightly formulated, is complete, I address the question of whether the liar sentence might alternatively be considered as biconditional in logical form, for the benefit of anyone (like Barker, as it turns out) who might mistakenly think that the liar sentence should be construed as the biconditional (L*) rather than as the conditional (L).

4.

Barker remarks that: ‘The reason (L) fails to generate a contradiction is that it is simply too weak.’ I agree entirely with this. (L) is indeed too weak to entail a logical inconsistency. What I say is that if (L) is the right way to interpret the informal statement of the liar sentence, then it follows, as I believe and as I argue in my paper, that the liar sentence is incapable of supporting the liar paradox as an outright syntactical inconsistency.
5.

Where Barker begins to go disastrously wrong is in his next sentence, when he states: ‘In fact, (L) would be satisfied by substituting any false sentence for \( L \) whatsoever. In that case, (L)’s antecedent would be false, making (L) itself true. For example, we could substitute ‘Elephants are smaller than gerbils’ for \( L \) in (L) and get the true material conditional (1). If elephants are smaller than gerbils, then “Elephants are smaller than gerbils” is false. Since (1) has a false antecedent, it is a true sentence. So in effect, all that the condition (L) does is to take a sentence letter, \( L \), and declare it to be false; unsurprisingly, this does not lead to any sort of trouble.’

I agree that the material conditional in (1) is classically true since the antecedent is materially false. However, it certainly does not follow from Barker’s example that all that condition (L) does is to declare that sentence \( L \) is false. Rather, sentence \( L \) is self-referentially defined by (L) as a sentence that says of itself that it is false. \( L \), affirming its own falsehood, is evidently very different from the elephants-gerbils example that Barker tries to float. Once again, I agree entirely with Barker when he says shortly thereafter that: ‘To be a Liar sentence, a sentence must actually state its own falsity — or more exactly, for \( S \) to be a Liar sentence, \( S \) must actually be equivalent to the statement that \( S \) is false.’ This is precisely what I believe my original definition of sentence \( L \) accomplishes by condition (L). What is clear from what follows in Barker’s discussion is that he conflates the requirement that a liar sentence be equivalent in meaning to the statement that the sentence itself is false with the requirement that therefore the liar sentence must be formulated as a biconditional, a crucial claim for which Barker offers no argument whatsoever.

Now, it is perfectly true that a logical equivalence must be formulated as a biconditional. However, not every sentence that is logically equivalent to this or that meaning or to this or that other sentence must itself be formulated as a biconditional. This important and relatively elementary distinction seems to elude Barker. Thus, the sentence ‘All triangles have three sides’ is logically equivalent to the sentence ‘All squares have four sides,’ since both sentences are logically, or, perhaps, analytically true; yet neither of the sentences is itself biconditional in logical form. Universals are generally formalized by means of the conditional, here as \( \forall x[\text{Triangle}(x) \rightarrow \text{Three-sided}(x)] \) and \( \forall x[\text{Square}(x) \rightarrow \text{Four-sided}(x)] \) — where, of course, the converse conditionals do not hold true, since three-sided unclosed geometrical objects are not triangles, and not all four-sided closed or unclosed geometrical objects are squares. As a consequence,
although I agree with Barker that the conditional liar sentence \( L \) is too weak to uphold the liar paradox, I find myself unreservedly in disagreement with Barker when he concludes without argument: ‘(L) is simply too weak to capture the idea that \( L \) is a Liar sentence.’

Apparently unaware of having fallen into these confusions, Barker continues: ‘Clearly, the intent of (L) was something more — (L) was intended to say not that \( L \) is false, but that \( L \) is a Liar sentence.’ That was indeed my intent. There are of course numerous ways to say that a sentence is false, but I, in company with all the authors I know of who have written responsibly on the paradox, take a liar sentence to state self-referentially of itself that it is false. The real question here is whether or not sentence \( L \) as defined by (L) accomplishes this purpose and lives up to this intent. Part of the problem is that Barker in criticizing my sentence condition (L) shifts his choice of vocabulary repeatedly throughout these key passages, and does not pause to define or carefully explain how he is using the relevant terms. We are told in one place that the liar sentence must ‘say’ that it is a liar sentence, that it must ‘actually state’ its own falsity, which, in the passage examined above, is said to mean that it is ‘equivalent’ to the statement that it is false. Barker indicates that for a liar sentence candidate to ‘imply’ that it is false is not enough to meet the bill, but he does not say why. He says that many sentences, including self-contradictions, imply that they are false without qualifying as liar sentences. This, of course, is perfectly true; but my conditional sentence \( L \) as defined by condition (L) does merely imply but explicitly states of itself that it is false, introducing sentence \( L \) as none other than the sentence that says sentence ‘\( L \)’ is false.

I agree absolutely with Barker that a liar sentence must state of itself that it is false. The question nevertheless remains as to precisely how this is to be done. I think that the conditional liar sentence (L) accomplishes this purpose, while Barker claims that we need the biconditional sentence (\( L^* \)). What Barker does not say or adequately support by argument is the reason why. Barker commits an obvious non sequitur when he argues thereafter: ‘A logically inconsistent sentence implies its own falsity, for example, simply because it implies everything whatsoever. But such sentences are not Liars. (L) is simply too weak to capture the idea that \( L \) is a Liar sentence.’ I agree, once again, that all logically inconsistent sentences imply their own falsity. Notice, however, that I do not advance (L) as a formulation of the liar sentence on the grounds that it is logically inconsistent; in fact, I argue explicitly that while \( L \) expresses the informal meaning of the colloquial liar sentence, it is not strong enough to imply a contradiction. How does it follow from what Barker says that (L) ‘is too weak to capture the idea that \( L \) is a Liar sentence’? It would fail to capture the logical force of the liar if the conditional formula were too weak for
this purpose, but that is precisely what Barker needs to show, and
precisely what he goes no distance at all toward showing.

6.

Next, Barker tries making an appeal to authority. He writes, just before
introducing the biconditional form of the liar sentence, following my
notation, as (L*): ‘The following is just about universally recognized as an
appropriate condition.’ The word ‘universally’ is tagged by Barker with an
endnote number, and I expected to find in the note at least some
documentation in support of the claim that ‘just about’ everyone except
myself recognizes the biconditional in (L*) as the correct formalization of
the liar sentence. The note, however, contains no such information. Barker
reserves any discussion of what other logicians and philosophers have said
about the proper way to understand the logical structure of the liar
sentence until the very last page, where he quotes Alfred Tarski’s ‘The
Concept of Truth in Formalized Languages’ and paraphrases Saul A.
Kripke’s essay, ‘Outline of a Theory of Truth.’ These are obviously names
to conjure with in recent discussions of the liar paradox, so I shall skip
ahead to this part of Barker’s argument.

What I find in Barker’s quotation from Tarski is support for rather than
any challenge of my conditional reading of the liar, and no justification at
all for the biconditional interpretation. Tarski begins with an at-first
unnamed sentence that says of sentence c that it is not a true sentence.
Then in (α) Tarski identifies the sentence in question as c, which is to say
that he names as c the sentence that says of sentence c that it is not a true
sentence. As I read Tarski, c, which is obviously not a biconditional, is
just the liar sentence. Proposition (β) that follows in Barker’s quotation of
Tarski, in turn, is not the liar sentence, but the liar paradox which is
supposed to depend logically on the liar sentence. Tarski’s (β) states: “‘c
is not a true sentence’ is a true sentence if and only if c is not a true
sentence.’ That is, Tarski’s (α) expresses the liar sentence, and (β)
expresses the liar paradox resting on (α). The liar paradox in Tarski’s (β)
is biconditional, just as we would expect it to be, but the liar sentence, in
Tarski’s (α), and as I would also insist, is conditional rather than
biconditional. Barker, however, has a different interpretation. He
continues: ‘Tarski then goes on to explain how (β) yields a contradiction
when combined with an unrestricted version of [the Tarskian truth
schema] (TS). (β) is, of course, simply a substitution instance of [a
biconditional formulation of the liar sentence].’ Clearly, however, Tarski’s
(β) does not need to ‘yield a contradiction’ when combined with anything
at all, since (β) itself is already most explicitly a contradiction. Nor does Barker continue to quote a relevant passage from Tarski in which this further inference of a contradiction amounting to the liar paradox is supposed to be derived. Proposition (β), I should say, as I understand Tarski, is already and in full his statement of the liar paradox, the contradiction that is supposed to follow from the constructability in a colloquial language of the liar sentence defined in (α) when sentence \( c \) is identified self-referentially as the sentence that says of itself that sentence \( c \) is not a true sentence. Naturally, the liar paradox, if it can be derived, will be formulated as a biconditional, as we find it presented in Tarski’s (β); but that is not the liar sentence, which appears in (α), as I would urge, as a conditional, or at least without any explicit biconditional. Do we then need to interpose a biconditional in formalizing Tarski’s liar sentence (α)? Barker does not try to argue the point, but seems to have simply confused Tarski’s conditional liar sentence with his biconditional statement of the liar paradox. I think, on the contrary, that Tarski’s liar sentence (α) must be expressed as a conditional rather than as a biconditional, which I would formulate conditionally in this way:

\[
c: \forall x [x = c \rightarrow x \text{ is FALSE}].
\]

Note that we cannot reasonably adopt a counterpart biconditional formulation of the sentence, for then we would be identifying \( c \) as the sentence stating that every false sentence is identical with sentence \( c \). Tarski obviously intends no such thing; and rightly so, since it would wrongly identify a particular sentence \( c \) with each and every distinct false sentence.

Scouring Kripke’s landmark essay, ‘Outline of a Theory of Truth,’ Barker is unable to come up with even a single quotation that remotely supports his claim that the liar sentence as opposed to the liar paradox is properly formulated as a biconditional. Instead he offers the following synopsis: ‘Similarly, …Kripke shows how to construct self-referential sentences in several different ways, using the techniques outlined above. [The biconditional liar sentence] is never derived explicitly, because there is no need: the existence of such sentences is enough to provide a contradiction in the presence of (TS) and classical logic.’ I am not sure I understand just what Barker is trying to say here, but what stands out is his true-enough claim that Kripke does not offer an explicit derivation of a biconditional liar. Barker remarks that the existence of ‘such [self-referential] sentences’ in Kripke’s essay is enough to provide a contradiction, but (a) the issue here is exactly what logical form such self-referential sentences must have, and Barker does not provide enough information to determine whether Kripke regards self-referential truth denials as biconditional rather than conditional in logical form; moreover
(b) merely affirming that ‘the existence of such sentences is enough to provide a contradiction’ begs the question as a criticism of my analysis of the liar if the objection is supposed to be that my analysis fails to constitute a genuine paradox when the liar sentence, as I continue to believe, is properly formulated as a conditional rather than as a biconditional.

I find it ironic, then, to discover Barker concluding this section of his critique by writing: ‘Overall, I am aware of nothing in the literature that could be considered a “conditional Liar” in Jacquette’s sense of the term.’ The scant references to other writers that Barker offers are equivocal at best, and go no distance at all toward showing that the liar sentence as opposed to the liar paradox must be properly formulated as the biconditional (L*) rather than as the conditional (L). I do not personally doubt that there might be respected authors who hold that view, although at the moment I do not happen to know of any. I addressed my original paper to a mainstream of writers on the subject whom I suspect to have at least implicitly adopted what I continue to regard as this mistaken assumption about the logical structure of the liar sentence. However, I am surprised that Barker, who has the courage to say, concerning the main bone of contention between us in his critique of my argument, that the biconditional liar sentence in (L*) is ‘just about universally recognized as an appropriate condition,’ is not able to put forward any more convincing documentation of this alleged pressure wave of opinion than his misreading of Tarski and inconclusive paraphrase of Kripke.

More to the point, since philosophical issues are decided by good reasoning rather than opinion polls, Barker would need to discover and defend sound arguments, not just amass contrary statements of commitment, to show that the liar sentence must be formulated as a biconditional rather than as a conditional. Barker would also need to show that these arguments constitute an effective refutation of the objections I have raised here in this reply and in the Addendum to my original paper against interpreting the liar sentence as opposed to the liar paradox as a biconditional. This is a palpable burden of proof that Barker does not even to try to meet, and without succeeding at which the reader has no grounds for accepting his repeated unsupported insistence that the liar sentence must be biconditional.

To recap my brief against biconditional formulations of the liar sentence from the Addendum to my original paper, I maintain that it would be logically uninteresting to present the liar sentence as a biconditional, because to do so is tantamount to simply stating an outright syntactical inconsistency of the form $p \iff \neg p$. The fact that a language permits the construction of any formal contradiction does not show that the language is internally inconsistent. It is only if the liar sentence, while
falling short of an open contradiction in its very formulation, nevertheless entails a logical inconsistency, that we can meaningfully speak of a genuine logical paradox about the constructibility of sentences that deny their own truth. Barker says nothing whatsoever to counter my Addendum objection to nontrivially formulating the liar sentence as a biconditional. In what I still consider to be the only proper, potentially logically interesting, conditional formulation of the liar sentence, as my original paper shows, and as Barker also admits, the liar sentence is too weak validly to sustain the second FALSE → TRUE horn of the liar paradox.

7.

The liar sentence says, informally: ‘I am false.’ This, I maintain, is rightly understood as a conditional assertion. The liar sentence clearly does not say, biconditionally, as Barker apparently would have it: ‘I am false if and only if I am true’ or ‘I am false if and only if it is not the case that I am false.’ Any such formulation of the liar sentence would obviously make the liar paradox built on the sentence altogether trivial.

In the case of the aboriginal Epimenides paradox in the Gospel of St. Paul, the proto-liar sentence says only conditionally, as spoken by a Cretan, (C): ‘All Cretans are liars.’ Interestingly, when we try to superimpose the liar argument on this informal version of the liar sentence, we do not collect a full paradox from both dilemma horns. If (C) is true, as with (L), then it is false; whereas, if (C) is false, then it is only possibly true. It follows in that case that there is at least one Cretan who is not a liar; but the veracious Cretan need not be Epimenides as the particular Cretan who utters (C). It will not do to argue that if (C) is false, then Epimenides as a Cretan by virtue of having lied has spoken truly in asserting that all Cretans are liars, because logically the falsehood of (C) does not exclude the possibility that Epimenides does not lie on all occasions, let alone that all other Cretans always lie. He might or might not be telling the truth when he says falsely that all Cretans are liars. This is a limitation of the original liar sentence, which, consistently with my analysis, supports only half of the liar paradox in the first paradox dilemma horn. It is something that experienced logic teachers learn to dance around when presenting the liar paradox historically by way of the Epimenides.

If we switch to a biconditional reformulation of the Epimenides, (BC), saying that ‘All and only Cretans are liars,’ then, significantly in light of Barker’s objections to my conditional formulation of the liar sentence, the situation logically is not improved. In that case, the falsehood of (BC), its
not being the case that all and only Cretans are liars, implies that there exists at least one individual who is either a liar but not a Cretan, or a Cretan but not a liar. Even if we suppose that Epimenides was the only Cretan, which is neither factually true nor guaranteed by logic, it still would not follow logically from the falsehood of (BC) that Epimenides in particular was not a liar — only and at most that some Cretan was not a liar or that some liar was not a Cretan. As a consequence, it does not follow even for the amped-up version of Epimenides’ statement, holding biconditionally that all and only Cretans are liars, that (BC) must turn out to be true on the assumption that it is false.

8.

Barker goes to some length to prove that the biconditional (L*) entails a logical inconsistency, but I do not understand why he bothers. I already made an argument to this effect in the Addendum on the Biconditional Liar in my original essay. It is undeniable that we can validly derive a contradiction from the biconditional liar sentence in a classical bivalent semantic environment, provided we begin ill-advisedly with a formulation such as (L*) \( \text{L* FALSE} \). (L*).

9.

I am further puzzled by Barker’s later admission that: ‘Granted, it is quite true that if we posit (L*), then in light of the standard truth schema, we are effectively assuming a contradiction. That is exactly the point: (L*) is contradictory in light of the standard truth schema, which demonstrates that there is either something wrong with the standard truth schema, or something wrong with (L*). The Liar paradox, as standardly understood, is precisely this conflict between the standard truth schema and (L*). Either one has to go, or the other does.’ What baffles me is Barker’s dilemma.

Of course, I have no hesitation in saying that (L*) must go, recognizing as I have said repeatedly that it trivially amounts to the statement of an outright logical inconsistency and hence as a defectively uninteresting basis for the derivation of a genuine paradox. Barker, on the other hand, is committed to (L*), and, as we shall see in item 10 immediately below, he actually claims to be able to deduce (L*) as a substitution instance for a theorem of Peano arithmetic! This leaves him only with the possibility of rejecting Tarski’s truth schema. What I would say is that Tarski’s truth
schema is the most innocent party in all this mess, and that if anything we must preserve the truth schema as intuitively well-justified, and instead reject \((L^*)\). If \((L)\) formally entailed the liar paradox as a logical antinomy, as it appears in its standard informal exposition to do, then we would be in serious trouble, and perhaps we would need to consider rejecting either the truth schema or classical logic. The argument I offer in my original paper is that the conditional sentence \(L\) defined by \((L)\) is the only legitimate formulation of the liar sentence, and that \(L\) is too weak to produce a syntactical inconsistency in the second paradox dilemma horn to support the derivation of the liar paradox. Informally, it appears that there is an interesting liar paradox based on the meaning of the liar sentence, but on a more searching formal analysis the paradox vanishes.

10.

I am even more astonished thereafter to read the following assertion in Barker’s criticism of my essay. He begins by asking: ‘So why not simply deny \((L^*)\)? Wouldn’t that be better than denying \((TS)\)? It would be, except that we are stuck with \((L^*)\): it is provable. More specifically, \((L^*)\) has substitution instances that are mathematically provable. It also has substitution instances that are empirically verifiable. These facts about \((L^*)\) are quite well known, but let us briefly review them here.’

What follows in Barker’s essay, insofar as I can make sense of it, is mistaken on every count. He writes: ‘First, we can construct a substitution instance of \((L^*)\) mathematically. For any formula \(F(x)\) of the language of arithmetic, we can find a sentence \(A\) such that the following is a theorem of Peano arithmetic: (A) \(A ⇔ F^{A}_A\). This is Gödel’s famous Diagonal Lemma, and it does not essentially depend on what formal language we work in; it continues to hold if we add extra predicates (say) to the language of arithmetic, and in that case \(F\) may be any formula of the extended language.’ Here I must assume that Barker has somehow misstated himself, since he first says that \(F(x)\) is a formula, and then that \(F\) is a formula; but if \(F(x)\) is a formula, then \(F\) itself is undoubtedly a predicate. Barker continues: ‘In particular, if we add the unary predicates \(TRUE\) and \(FALSE\) to the language of arithmetic, then as a special case of (A) we have, for some sentence \(L\), \((A') L ⇔ FALSE^{\left[L\right]}\). I emphasize that the sentence \((A')\) is a theorem of Peano Arithmetic, and is in no way an extra posit. \((A')\) is also, of course, formally identical to \((L^*)\), and so Peano Arithmetic would be rendered inconsistent by the addition of the premises \((CL)\) and \((TS)\).’

Of course, Peano arithmetic, historically at least, if we are talking about the Dedekind-Peano axioms, already presupposes \((CL)\), while \((TS)\), as I
suggested above, seems intuitively the most harmless of additions or presuppositions. It follows that Barker, on the basis of the argument above, regards Peano arithmetic not only as deductively incomplete, which is something more closely akin to Gödel’s own conclusion about A.N. Whitehead and Bertrand Russell’s Principia Mathematica (‘und verwandter Systeme’) in his 1931 paper, but actually logically, syntactically, inconsistent. Something is evidently dreadfully wrong here.

It is worth noting that Barker interprets his corner quotes in a rather special way, entirely different than my own. His endnote 2 reads: ‘I am assuming a fixed Gödel numbering as given, and treating $[A]$ as shorthand for the standard numeral of $A$’s Gödel number.’ In that case, $A$ is a sentence and $[A]$ is its Gödel number. However, I was not using the corner quotes in anything like this way, but merely as intensional Quinean quotes to mention a specific sentence to which the predicates TRUE and FALSE, among others, might be applied. Thus, when Barker takes my formula \((L^*)\) as a substitution instance of \((A')\), as he says above, he is already committing a very irregular, and, I would say, fallaciously invaliding, equivocation.

Moreover, what Barker proposes as a consequence thereafter is manifestly false. If \(F(x)\) is any formula of the language of arithmetic, and \(F\) any predicate of such a language (or extension thereof, as Barker allows), then let \(F\) be the predicate of being numerically odd and \(F(x)\) an open sentence that says that \(x\) is or has the property of being numerically odd. Turning back to \((A)\), we have \([A]\) as the Gödel number of sentence \(A\) in \(A \leftrightarrow [F[A]]\). Now suppose that in a fixed Gödel numbering, as Barker proposes, the Gödel number of sentence \(A\) is indeed odd. If, however, sentence \(A\) happens to be a false sentence, then \((A)\) will not be a theorem of Peano or any other arithmetic. The same objection can be worked in a variety of ways. Suppose that \(A\) is false, but its Gödel number is even, and \([F[A]]\) says truly that the Gödel number of \(A\) is even. Or, suppose that \(A\) is required to be a theorem of a sound arithmetic, and hence a true sentence. Still, \(A\)’s Gödel number in some fixed numbering in principle might be either even or odd. Since \(F(x)\) according to Barker can be any open sentence of the language of arithmetic, and \(F\) can be any predicate of the language, \((A)\) turns out to be false, and hence no theorem of Peano arithmetic, if Gödel number \([A]\) is odd, but \([F[A]]\) says that \([A]\) is even, or if \([A]\) is even, but \([F[A]]\) says that \([A]\) is odd. Similarly for an indefinitely large choice of arithmetical predicates that might truly or falsely apply to a given Gödel number for a sentence, including any theorem, of Peano arithmetic.

I have some familiarity with Gödel’s arguments, but I do not recognize anything as bizarre as Barker’s application of the ‘Diagonal Lemma’ in Gödel’s logical-arithmetical metatheorems. I think that Barker minimally
owes the reader a quotation from the relevant section of Gödel’s essay, or at least a reference to the passage on which he supposedly relies, without which it is impossible to understand what he is talking about or whether what he is saying bears any remote resemblance to Gödel’s result. If Gödel’s ‘famous’ lemma, as Barker characterizes it, entailed that all of Peano arithmetic, couched in classical logic and supplemented merely by a Tarskian truth schema, was out-and-out logically inconsistent, instead of being either syntactically inconsistent or deductively incomplete, then I think that Gödel’s lemma and his proof would be even more famous than they are. I can only conclude that Barker has seriously garbled the content and misunderstood the implications of Gödel’s 1931 paper, just as I think he has misunderstood and misapplied Tarski’s 1935 and 1956 statements of the liar sentence and liar paradox.

11.

Barker next moves to support his claim that there are also empirically verifiable substitution instances of the biconditional liar. He argues: ‘The other way to find a true substitution instance of (L*) is to construct a sentence which, as a matter of empirical fact, says of itself that it is false. For example, suppose we introduce a new constant $c$ into our language and decide to use it as a name for the sentence FALSE($c$) — we can use $c$ as a name for anything we like, including the string of symbols FALSE($c$). Thus, we are entitled to assert (SR) $c = \text{FALSE}(c)$. (SR) is perfectly consistent, provided we do not also assume the disquotational schema (TS). However, (SR) logically implies FALSE($c$) $\leftrightarrow$ FALSE(\text{FALSE}(c)) which is simply a substitution instance of (L*). So again, we are stuck with (L*) and will have to look somewhere else if we want to reject a premise of the Liar argument.’

It may not be true to say as Barker does that we can use $c$ to name ‘anything we like.’ If we observe the usual restrictions on impredicative definitions, then (SR) is immediately ruled out as improperly formulated. Let that pass, however, since Whitehead and Russell’s ban on impredication is generally introduced as a solution to a certain class of logical paradoxes, and we are in the business of trying to understand whether and how such paradoxes arise when no preventative measures prevail. If (SR) is admitted as a permissible stipulation, is Barker correct to say that (SR) logically implies FALSE($c$) $\leftrightarrow$ FALSE(\text{FALSE}(c))? In his endnote 3, Barker explains that he derives the biconditional FALSE($c$) $\leftrightarrow$ FALSE(\text{FALSE}(c)) from (SR) together with the tautology, FALSE($c$) $\leftrightarrow$ FALSE($c$). The substitution at first glance looks clean enough, but there are several hidden difficulties. Barker once again does not
consistently follow the conventions on Quinean corner quotes that I adopt in my original essay. In my notation, any TRUE or FALSE predication attaches to a term for a propositional sentence enclosed in corner quotes. When Barker writes FALSE(c), he steps outside of my usage. He is free to do so, naturally, but then the relevance of his constructions for the arguments in my target paper comes into question.

What Barker should have written, in keeping with my way of expressing a truth value predication, is rather (SR*) \[ c = \text{[FALSE}_c]. \] This seemingly inconsequential difference in fact helps to highlight an important defect in Barker’s inference. For now the tautology from which Barker derives his biconditional sentence, FALSE(c) \( \leftrightarrow \) FALSE(\text{[FALSE}_c]), needs to be written as FALSE\[c\] \( \leftrightarrow \) FALSE\[\text{c}\]. When Barker substitutes in this reformulation \[\text{[FALSE}_c\]] for c on the right-hand side of the tautology, on the basis of (SR*), what he should obtain by syntax copy and paste is instead, FALSE\[\text{c}\] \( \leftrightarrow \) FALSE\[\text{FALSE}_c\]. This revised biconditional, contrary to Barker’s claim concerning his original derivation in his variant notation, as inspection reveals, is no longer a syntactically exact substitution instance of (L*). We may have also thereby strayed into unknown syntactical territory, suggesting that Barker’s (SR) revised in accord with the corner quote convention as (SR*) is not well-formed, in the absence of a plausible interpretation of the iterated corner quotes it requires.

As I have indicated, I do not use corner quotes in this way, and I have no immediate recommendation as to how such iterated corner quotes might be read. As I use the notation, \[\text{[FALSE}_c\] , unlike FALSE\[\text{c}\], does not represent a sentence, but something like the name or mentioning of a sentence. It seems to say something like the naming or mentioning of named or mentioned item c is false, but I do not know what this would be or what its semantics would involve. Namings and mentionings of sentences, in my mind at least, unlike named or mentioned sentences, do not have truth value; they are rather, just as Barker says, strings of symbols. We are only entitled to intersubstitute coreferential terms that appear on opposite sides of the ‘=’ sign salva veritate, and Barker’s substitutional derivation of the formula he claims to be an instance of (L*) as a result does not appear to be truth-preserving. The same is true if we now relax the requirements of my own corner quotation convention to allow for Barker’s variation involving parentheses. The symbol string \[\text{[FALSE}_c\] is equally not a sentence, but rather a naming or mentioning of a sentence or the characters that make up the sentence FALSE(c). What, then, is Barker saying when he writes FALSE(\text{[FALSE}_c\])? What is the force of attributing truth or falsehood to the completed context, \(\{\ldots\}\)? What are the parentheses doing in addition to the corner quotes, and what possible meaningful purpose do they serve?

For that matter, we must also wonder whether c as Barker defines it could possibly be a sentence, true or false. Insofar as I understand what
Barker seems to be proposing in his definition of $c$, $c$ is not a sentence at all, but the naming or mentioning of a sentence, and hence a syntax sequence lacking any truth value. I think that this is why he introduces it by means of the identity sign, ‘$=$’, rather than by means of the biconditional, ‘$\leftrightarrow$’, which, connecting syntax items lacking truth value, would not make sense here. If $c$ is not itself a sentence bearing truth value, true or false, then the extra-propositional syntax combination to which Barker appeals, $\text{FALSE}(c) \leftrightarrow \text{FALSE}(c)$, is not actually a tautology, but itself lacks any truth value. It is not a well-formed formula in that case and cannot intelligibly support the valid derivation of anything at all, whether by substitution or any other logically approved deductively valid method.

I am also not sure I understand the exact sense in which Barker describes this set of substitutions as constituting an *empirically* verifiable substitution instance of (L*). He says that it is possible ‘to construct a sentence which, as a matter of empirical fact, says of itself that it is false,’ but he does not explain how we are to go about confirming or disconfirming that (SR) is contingently true or contingently false. Perhaps the construction is meant to have something to do with our purportedly free choice in using $c$ to name the string of symbols $\text{FALSE}(c)$, but I must admit that I did not grasp how a construction proceeding from an alleged tautology was supposed to be empirical, or by what observations or experiments we are supposed to be able to determine that a given sentence says of itself as an empirical matter of fact that it is false.

A final peculiarity of Barker’s argument concerns its derivation from the supposed tautology, $\text{FALSE}(c) \leftrightarrow \text{FALSE}(c)$, even under his misappropriation of the corner quote notation and the questionable status of $c$. Absolve Barker for the moment of the above complications involving the definition of $c$ and the attempt to exploit substitution in deriving his biconditional from what is supposed to be a tautology.

Consider, for the sake of argument, the substitution Barker tries to make as deductively valid and the construction from which he begins as a genuine tautology. If we can correctly obtain $\text{FALSE}(c) \leftrightarrow \text{FALSE}(\text{FALSE}(c))$ by substitution from the ‘tautology’ $\text{FALSE}(c) \leftrightarrow \text{FALSE}(c)$, then we can equally well perform the operation yet again in the opposite direction by substituting $c$ for $\text{FALSE}(c)$ on the right-hand side of Barker’s biconditional sentence, $\text{FALSE}(c) \leftrightarrow \text{FALSE}(\text{FALSE}(c))$, bringing us back once again to the original tautology, $\text{FALSE}(c) \leftrightarrow \text{FALSE}(c)$. More directly, since a tautology is implied by anything, Barker’s biconditional also logically implies the alleged tautology from which he
proposes to derive the biconditional. It follows, assuming Barker’s (SR) throughout, that \([\text{FALSE}(c) \leftrightarrow \text{FALSE}(\text{FALSE}(c)^3)] \leftrightarrow [\text{FALSE}(c) \leftrightarrow \text{FALSE}(c)]\).

Now, it might be that Barker’s biconditional \(\text{FALSE}(c) \leftrightarrow \text{FALSE}(\text{FALSE}(c)^3)\) is in some sense logically equivalent to the tautology from which he purports to derive it; but then it surely cannot be logically equivalent to the biconditional liar sentence (L*). Logical equivalence is transitive, yet obviously the biconditional liar sentence (L*) is not itself a tautology nor logically equivalent to any tautology. If (L*) were logically equivalent to a tautology via its logical equivalence to Barker’s biconditional \(\text{FALSE}(c) \leftrightarrow \text{FALSE}(\text{FALSE}(c)^3)\), then, since (L*), as Barker maintains, is logically inconsistent, then, too paradoxically, not only would the ‘tautology’ \(\text{FALSE}(c) \leftrightarrow \text{FALSE}(c)\) also be logically inconsistent, but no logically inconsistent liar paradox could be validly derivable from (L*). The liar paradox, make no mistake, as opposed to the liar sentence, is logically false when expressed as a biconditional, and no logical falsehood is validly deduced from a tautology. All this, we recall, moreover, according to Barker, where the biconditional liar sentence (L*) is provable as a substitution instance of a theorem of Peano arithmetic and stands as an empirically verifiable logically contingent sentence, is the fault exclusively of Tarski’s truth schema!